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Chaos Analysis on Librational Control of Gravity-Gradient Satellite in Elliptic Orbit

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I. Introduction

LIBRATIONAL motion of a gravity-gradient satellite[1] in an elliptic orbit is governed by a system of nonlinear equations that become nonautonomous when the effect of orbital eccentricity is taken into consideration. It is well known that chaotic phenomenon can be observed in such a nonlinear and nonautonomous dynamic system over some ranges of the initial conditions. With advancement of mathematical and computational tools, chaos has been analyzed as a nonlinear phenomenon by such techniques as Poincare maps, Lyapunov exponents, reconstruction of attractors, and bifurcation diagrams.[2–4] Karasopoulos and Richardson[5,6] have applied these techniques to analyze the nonlinear dynamics of the gravity-gradient satellite system. Nixon et al.[7] and Fujii and Ichiki[8] have also applied these techniques to analyze the nonlinear dynamics of the tethered satellite system.

Recently, a few control concepts for chaos have been developed to convert chaotic behavior into a periodic or constant motion. Pyragas has proposed a control method called time-delay feedback control (DFC) in Ref. 9. This method is robust to small parameter variations and has been successfully used in several applications. Various modifications have also been considered in Ref. 10 for stabilizing stronger chaotic systems. Analysis of the system stability controlled by DFC has been discussed in Refs. 11 and 12. The main contribution of the present paper is an attempt to apply the DFC scheme successfully to the nonlinear dynamics of gravity-gradient satellites. It is shown that the control scheme is extremely effective for the present chaotic system.

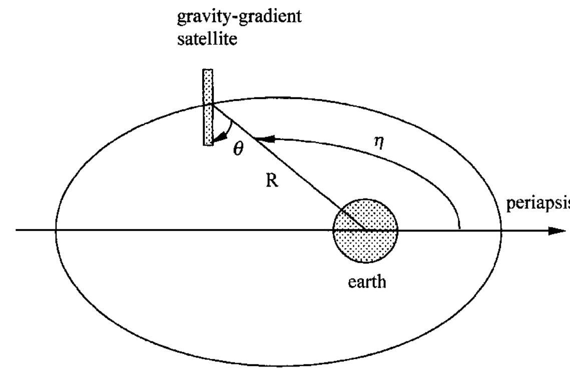

Fig. 1 System model

II. Dynamical Model

The dynamic system treated in this paper is a rigid gravity-gradient satellite in an elliptic orbit as shown in Fig. 1. The dynamic model is simplified by employing the following three assumptions:

1) Energy dissipation effects such as the aerodynamic drag will be ignored.

2) Only the pitch motion in the orbital plane is considered.

3) The inertia ratio of the satellite system is taken to be one, that is, the equal moments of inertia about the pitch and roll axes are significantly large compared to that about the yaw axis.

The equation of pitch motion for the rigid gravity-gradient satellite in an elliptic orbit can be described as follows:

$$\frac{d^2\theta}{d\eta^2} = -\frac{3}{2(1 + e\cos\eta)}\sin(2\theta) + \frac{2e\sin\eta}{1 + e\cos\eta}\left(\frac{d\theta}{d\eta} + 1\right) - u(\eta) \tag{1}$$

where, θ, e, and $u(\eta)$ denote the pitch angle, the orbital eccentricity, and the control input, respectively, and the independent variable is the true anomaly η.

In general, chaotic motion means that the behavior of the system can be predicted for the short term but not all of the time, and the motion is regarded as chaotic if it simultaneously satisfies the following two characteristics. The first characteristic is sensitive dependence on the initial conditions. Chaos occurs in the deterministic system, but exhibits random behavior, that is, trajectories of a chaotic system starting from two nearly initial conditions will eventually separate and become uncorrelated, but always remain bounded in space. The second characteristic is topological transition. The trajectory can be close to any points in the bounded space, and such a trajectory is said to be dense. Consequently, chaos can be defined as trajectories that are neither equilibrium points nor periodic ones in a bounded space.

Concerning our dynamic system, the eccentricity can affect the nonlinear dynamics through the coefficients of nonlinear terms, and chaotic motion may occur in the situation when the eccentricity is taken into account, as in Eq. (1).

III. DFC

The two fundamental characteristics of chaos mentioned earlier are used for controlling the chaotic system. By the use of the characteristic of sensitive dependence on the initial conditions, it is possible to have large influence on the system dynamics by very small perturbations or external influence; moreover, the response of the system

Received 24 March 1998; revision received 6 July 1999; accepted for publication 26 July 1999.

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becomes very fast. The important characteristic is the existence of an infinite number of unstable periodic orbits, which is the core phenomenon of chaos. This structure of unstable periodic orbits plays an important role for controlling chaos because wide choices exist to stabilize a chaotic orbit to one of the embedded unstable periodic orbits.

Controller Design

The DFC uses a delayed copy of the output signal.[9,10] This method to control the chaotic system has two positive features. First, it is a continuous self-feedback control without requiring preliminary calculation of the desired periodic, orbit other than its period, and DFC provides a natural choice for controlling chaos. Second, it does not require precise knowledge of the system parameters and is robust to small parameter variations.

The control input $\mu(\eta)$ is chosen in the following form of the difference between the actual and T time-delayed output signal:

$$u(\eta) = K\{\dot{\theta}(\eta) - \dot{\theta}(\eta - T)\} \tag{2}$$

where K and T denote the constant feedback gain and the period of the desired unstable periodic orbit, respectively. The main feature of this input is that it vanishes if delay time T coincides with the period of an unstable periodic orbit for the system with the eccentricity zero. It means that the input does not change the solution of Eq. (1) corresponding to the unstable orbit with the period T, but does change the chaotic motion to the periodic one.

The linearized form of Eq. (1) around a periodic orbit is considered in the stability analysis of the controlled system that uses the DFC input of the form shown in Eq. (2). The linearized equation can be written as

$$\dot{\Theta}(\eta) = \sum_{i=0}^{1} A_i(\eta) \cdot \Theta(\eta - iT) \tag{3}$$

$$A_0(\eta) = \begin{pmatrix} 0 & 1 \\ -\dfrac{3\cos(2\theta_p)}{1 + e\cos\eta} & \dfrac{2e\sin\eta}{1 + e\cos\eta} - K \end{pmatrix}, \qquad A_1(\eta) = \begin{pmatrix} 0 & 0 \\ 0 & K \end{pmatrix} \tag{4}$$

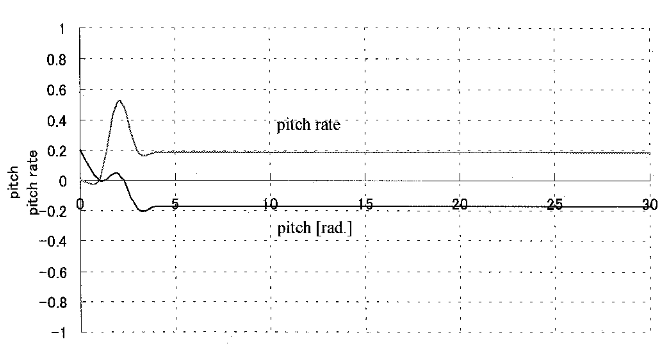

Fig. 2a Response of pitch and pitch rate at $e = 0.36$ and $K = 1.0$.

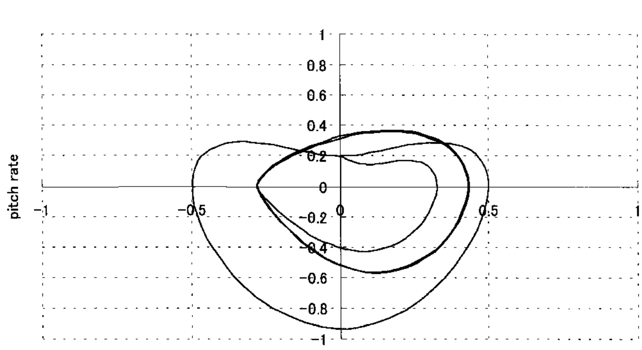

Fig. 2b Phase space at $e = 0.36$ and $K = 1.0$.

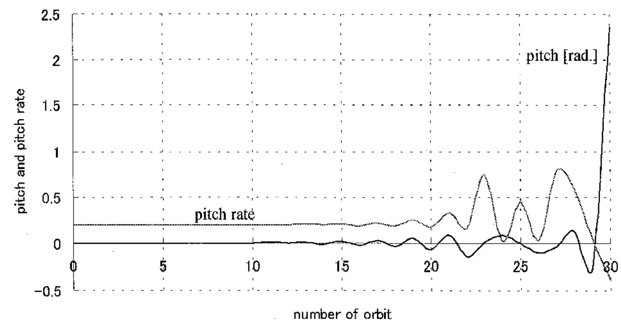

Fig. 3 Uncontrolled pitch and pitch rate at $e = 0.36$.

where θ_p is the periodic solution of Eq. (1) for θ, with a period of one orbit and $A_0(\eta)$, $A_1(\eta)$, and

$$\sum_{i=0}^{1} A_i(\eta)$$

are periodic with period T. Note that the solution of linear equations with time-varying coefficients of period T can be obtained by the Floquet theory.

Numerical Results

The DFC has been demonstrated through numerical simulation to stabilize the gravity-gradient satellite system to one periodic orbit for the case of $e = 0.36$ with the initial condition at $(\theta_0, \dot{\theta}_0) = (0.0, 0.1985)$ and the feedback gain at $K = 1.0$. The time responses of the controlled pitch and pitch rate and the corresponding phase space are shown in Figs. 2a and 2b, respectively. Control input is implemented after the time period of one orbit. The response of the uncontrolled pitch and pitch rate are shown in Fig. 3 for the sake of comparison. The result shows that DFC is extremely effective for our dynamic system in controlling the chaotic motion.

IV. Conclusions

Chaotic librational motion of a gravity-gradient satellite in an elliptic orbit is controlled by using the DFC scheme. DFC is shown to be particularly effective in controlling the chaotic motion of the present system by appropriate utilization of the fundamental characteristics of chaos.

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